Engineering Calculus 1
Mini Project 4: Science Communication

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This project will challenge you to fix someone else's mistakes and then communicate what problems you encountered and how you solved them. It may happen that you get hired after someone else started a project, or that you have to work in a team with members who are less knowledgeable than you. You should be polite but firm when you make corrections and convince the higher ups (who may not have your level of expertise) that your approach is, in fact, the right one.

You have a week to complete this project. Do not wait until the last day. The sooner you start, the more opportunities you will have to ask for help if you need it. Reach out if any of the instructions are not clear or you would like any feedback.

Before We Begin...

You can use any of the tools made available in previous projects if they make your job easier. The key objective is to figure out a good strategy to solve problems and then fully justify your work to a non-expert. When in doubt, feel free to discuss with classmates or reach out for help.

When you present any part of the work assisted by technology, **make sure to include the exact formulas and settings you used. Your results should be reproducible.**

Your deliverable will be a slideshow presentation with at most 15 slides. It's okay if some are "repeated" by any transitioning effects, think about it as 15 distinct slides. They should tell a good story: explain where you started, what decisions you made, and why you are confident that you solved the problems.

1 Storyline

WME (We Make 'Em) is a goods manufacturing company. They make all sorts of products in honor of the founder, who when asked if they offer a particular item would always respond "We make 'em!"

There are different product lines you will be in charge of:

1. **Bedsheets.** These are produced year round in several batches of equal size and have constant demand throughout the year.
2. **Pumpkin-scented candles.** These are seasonal. Demand peaks in early October each year.
3. **Pet accessories.** New designs are released in batches of different sizes every month. Every once in a while pictures of a design go viral on social media and demand spikes.

All you know about the person in charge before you is they no longer work for the company (so reaching out for explanations on previous work done is not an option). You have to catch up with progress made so far and then complete the work (after making any necessary corrections).

2 Production and Storage

When producing and selling bedsheets there are two things to consider: the startup costs of getting the machines running when producing a batch and the storage costs. Costs can be split into fixed costs which do not depend on the quantity of items produced (keeping the lights on, paying workers) and variable costs (materials, utilities). In the work that follows, we will assume the following variables are measured in thousands

$$\begin{aligned} q &= \text{ number of units in each batch} \\ f &= \text{ fixed setup cost to manufacture the product} \\ g &= \text{ cost of manufacturing a single unit of the product} \\ k &= \text{ cost of storing one unit of the product for one year} \\ M &= \text{ total number of units produced annually.} \end{aligned}$$

The previous manager left instructions to make an entire year's worth of product and pull it out of storage as needed.

1. What does this strategy minimize?
2. Run some numbers (choose values for the variables). Is this always the best strategy? Propose a few different options and compare the resulting total manufacturing cost and total storage cost.
3. The total production cost, $T(q)$, is the sum of the manufacturing cost and the storage cost. Find an expression for $T(q)$ in terms of q, f, g, k and M. When is this minimized?
4. From the previous manager's notes:

   > The function $T(q)$ is a marginal cost. The overall costs can be obtained by adding the values of $T(q)$ over the years. The overall costs a years after starting production can be computed as
   >
   > $$\int_0^a T(q)dq.$$

   Do you agree with this analysis? If so, compute the integral. If not, explain how you would use $T(q)$ to obtain an expression for the overall costs incurred on the bedsheet line.
5. The bedsheet line is one of the oldest in the company, it has been in existence since 1982. Let t represent the number of years since 1982 and assume the average inflation rate is 3.8% per year. How would you find the overall costs if you use the value M from 1982 but account for inflation in all other costs? Is this a realistic estimate? Discuss ways to make a better approximation.
6. For historical reasons, in economics, the demand (as the price consumers are willing to pay for an item) is often written as a function of quantity. That is, we write $p = D(q)$. In this expression, which one is the dependent/independent variable?
7. Sketch $D(q)$ on a graph for the bedsheet line. Make sure to label your axes in a manner that is consistent with your answer above.
8. Write an expression for the revenue from the bedsheets on a particular year. How would you adjust this to account for inflation? Use this to compute the overall profit, a years after starting production.

3 Seasonality

9. Data for the pumpkin candles is collected daily. It is known that every year, sales peak at around 25,000 units sold on October 1st. By December 1st, sales will have dropped to about 19,000 units. When you open up the file that models this behavior you see this function:

$$a(t) = -1.5(t-9)^2 + 25$$

   There were no notes or comments to explain the reasoning. Can you figure out what t represents?

10. Plot the function. Is this a good function to model the data? Can you think of a better alternative?

11. On a spreadsheet you see that average sales from August 1st to December 1st are about 23,000 candles per day. Is this consistent with the previous manager's model? With your model? What do the two models say about average sales from the other thirds?

12. Use your proposed alternative to find a function to describe the total sales since January 1st on a given day of the year. This is nowhere to be found in the old manager's notes, but just seems like a good thing to have.

13. Assuming the behavior is the same every year, how many candles should you produce in total in a year?

14. If each candle costs $4 to make, store and ship, how much should each candle sell for in order to make a profit of $50,000 in a year?

4 Sudden Change

Pet accessories are especially hard to work with. You can decide to either launch a new design or keep the last month's design and you can choose how many units to make each month. However, you don't have control or any good way to predict demand. All you can do is react to changes in demand and plan a month at a time accordingly.

The only information we have available from the economists who study fads is that they tend to last anywhere between one and six months, and the faster they grow in popularity, the more rapidly they fall out of fashion as well.

The previous manager was able to find a roughly linear increase in sales after each release, and planned repeat orders of popular designs by multiplying the previous order's quantity by the slope. That is, if each day about 2,000 more accessories were sold, the next order would be twice as big as the previous one. However, historically this led to overproduction and a lot of inventory that had to be marked down significantly.

15. What is the variable here? What does the slope of the linear pieces represent?

16. After playing around with the sales data for a bit you found that when sales spike, this spike is roughly linear so it can be approximated by a function like

$$\begin{cases} 1-|x| & |x| \leq 1 \\ 0 & |x| > 1 \end{cases}$$

   What is missing here is something to show how some sales increase faster than others.

   Empirically, it has been observed that for the pet accessory line the length of time during which sales increase is inversely proportional to the maximum daily sales.

Let N be a novelty factor of a design. In the model, N corresponds to the slope of the linear pieces. Adjust the function above to incorporate N and the inverse relationship it has with the duration of high sales.

17. As you look through old data you find some products that spiked faster (and higher) than others, so you get a feel for just how much this novelty factor can vary. What is not clear just yet is how it affects overall sales, as all the data you can find consists of daily sales. The previous manager did this for a couple of best sellers by painstakingly adding up all sales: 18,062 for a design with bones, 17,894 for a design with polka dots in the pantone color of the year, 18,201 for the design with stripes in primary colors. However, that doesn't seem like the right way to go. There should be a way to quickly estimate total sales for a launch using the function above. Is the fact that you have no control over the novelty factor a problem? Explain.

18. Can you explain the trends in total sales?

19. What should the production strategy be moving forward?

20. Every once in a while, a design is so good it becomes a classic. Unlike fashionable designs, which have a finite "shelf life," classics can continue to be sold throughout the year, even if they start out as trendy items. How would you identify classics from daily sales data? Are all classics worth keeping? Explain.

5 Thinking Ahead

You found an old business plan proposal by the previous manager. After t years of making the proposed changes in manufacturing, the calculations predicted a rate of savings

$$S'(t) = 64 - t^2$$

and a rate of costs

$$C'(t) = t^2 + 28t$$

measured in thousands of dollars. Though t is measured in years, this function can be used to approximate savings and costs on a daily basis, so assume these are continuous (and not discrete) functions.

21. What do you think was the manager's reasoning when proposing this plan?

22. Was this a good plan? How long would this plan work to save money?

23. How much money would be saved overall?

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Rubric

	1	2	3	4	5
Prolbem-solving	All answers are missing or fail to address errors by the previous manager.	A majority of answers are missing or major mistakes remain	Some answers incorrectly address the problems	Most answers correctly fix the errors	All answers correctly fix the errors
Calculus concepts	The work shows serious misunderstandings when using calculus concepts	The work shows some misunderstandings when using calculus concepts	The explanations given are generally correct but incomplete	Most explanations are correct and complete	All explanations are correct and complete
Math work	Many solutions are incorrect or incomplete	A few solutions are incorrect or incomplete	Some solutions are missing steps or have small errors	A few solutions are missing steps or have small errors	All solutions are correct and complete
Analysis	All answers are left blank	A majority of answers are left blank or show very shallow analysis	About half of the answers are left blank or show very shallow analysis	Most questions are answered in depth	All spaces are answered in depth
Clarity	It is hard to read/follow the work	Some of the work is hard to read/follow	The organization/tidiness leaves room for improvement but is readable	The work is generally easy to read/follow	It is very easy to read/follow the work done
Explanations	Missing or very incorrect	Given but largely inappropriate (too shallow/technical) or incomplete	A mix of good explanations and not so great ones	Mostly good explanations: enough (but not too much) information	All good explanations.